Formulaire

Coordonnées cartésiennes :

$$\overrightarrow{\mathrm{grad}} f = \vec{\nabla} f = \begin{pmatrix} \frac{\partial f}{\partial x} \\ \frac{\partial f}{\partial y} \\ \frac{\partial f}{\partial z} \end{pmatrix}.$$

$$\mathrm{div}\vec{F} = \vec{\nabla} \cdot \vec{F} = \frac{\partial F_x}{\partial x} + \frac{\partial F_y}{\partial y} + \frac{\partial F_z}{\partial z}$$

$$\overrightarrow{\mathrm{rot}}\ \vec{F} = \vec{\nabla} \wedge \vec{F} = \begin{pmatrix} \partial F_z/\partial y - \partial F_y/\partial z \\ \partial F_x/\partial z - \partial F_z/\partial x \\ \partial F_y/\partial x - \partial F_x/\partial y \end{pmatrix}$$

$$\Delta = \nabla^2 = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2}.$$

$$\tilde{\Delta}\vec{A} = \tilde{\nabla}^2\vec{A} = \begin{bmatrix} \frac{\partial^2 A_x}{\partial x^2} + \frac{\partial^2 A_x}{\partial y^2} + \frac{\partial^2 A_x}{\partial z^2} \\ \frac{\partial^2 A_y}{\partial x^2} + \frac{\partial^2 A_y}{\partial y^2} + \frac{\partial^2 A_y}{\partial z^2} \\ \frac{\partial^2 A_z}{\partial x^2} + \frac{\partial^2 A_z}{\partial y^2} + \frac{\partial^2 A_z}{\partial z^2} \end{bmatrix} = \begin{bmatrix} \Delta A_x \\ \Delta A_y \\ \Delta A_z \end{bmatrix}$$

Coordonnées cylindriques :

$$\vec{\mathrm{rot}}\vec{A} = \left(\frac{1}{r}\frac{\partial A_z}{\partial \theta} - \frac{\partial A_\theta}{\partial z}\right)\vec{u_r} + \left(\frac{\partial A_r}{\partial z} - \frac{\partial A_z}{\partial r}\right)\vec{u_\theta} + \frac{1}{r}\left(\frac{\partial}{\partial r}(rA_\theta) - \frac{\partial A_r}{\partial \theta}\right)\vec{u_z}$$

$$\mathrm{div}\vec{A} = \frac{1}{r}\frac{\partial}{\partial r}(rA_r) + \frac{1}{r}\frac{\partial A_\theta}{\partial \theta} + \frac{\partial A_z}{\partial z}$$

$$\vec{\mathrm{grad}} f = \frac{\partial f}{\partial r}\vec{u_r} + \frac{1}{r}\frac{\partial f}{\partial \theta}\vec{u_\theta} + \frac{\partial f}{\partial z}\vec{u_z}$$

$$\Delta f = \frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial f}{\partial r}\right) + \frac{1}{r^2}\frac{\partial^2 f}{\partial \theta^2} + \frac{\partial^2 f}{\partial z^2}$$

Coordonnées sphériques :

$$\vec{\mathrm{rot}}\vec{A} = \frac{1}{r\sin\theta}\left(\frac{\partial}{\partial\theta}(\sin\theta A_\varphi) - \frac{\partial A_\theta}{\partial\varphi}\right)\vec{u_r} + \left(\frac{1}{r\sin\theta}\frac{\partial A_r}{\partial\varphi} - \frac{1}{r}\frac{\partial}{\partial r}(rA_\varphi)\right)\vec{u_\theta} + \frac{1}{r}\left(\frac{\partial}{\partial r}(rA_\theta) - \frac{\partial A_r}{\partial\theta}\right)\vec{u_\varphi}$$

$$\mathrm{div}\vec{A} = \frac{1}{r^2}\frac{\partial}{\partial r}(r^2 A_r) + \frac{1}{r\sin\theta}\frac{\partial}{\partial\theta}(\sin\theta A_\theta) + \frac{1}{r\sin\theta}\frac{\partial A_\varphi}{\partial\varphi}$$

$$\vec{\mathrm{grad}}f = \frac{\partial f}{\partial r}\vec{u_r} + \frac{1}{r}\frac{\partial f}{\partial\theta}\vec{u_\theta} + \frac{1}{r\sin\theta}\frac{\partial f}{\partial\varphi}\vec{u_\varphi}$$

$$\Delta f = \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial f}{\partial r}\right) + \frac{1}{r^2\sin\theta}\frac{\partial}{\partial\theta}\left(\sin\theta\frac{\partial f}{\partial\theta}\right) + \frac{1}{r^2\sin^2\theta}\frac{\partial^2 f}{\partial\varphi^2}$$

Relations :

$$\mathrm{div}(\vec{X_0}\wedge\vec{F}) = -\overrightarrow{\mathrm{rot}}\vec{F}\cdot\vec{X_0}$$

$$\vec{\mathrm{rot}}(\vec{\mathrm{grad}}) = \vec{0}$$

$$\mathrm{div}(\vec{\mathrm{rot}}) = 0$$

$$\vec{\mathrm{rot}}(\vec{\mathrm{rot}}) = \vec{\mathrm{grad}}(\mathrm{div}) - \vec{\Delta}$$

$$\Delta = \mathrm{div}(\vec{\mathrm{grad}})$$

$$\vec{\mathrm{grad}}(\vec{X_0}\cdot\vec{B}) = (\vec{X_0}\cdot\vec{\mathrm{grad}})\vec{B} + \vec{X_0}\wedge\vec{\mathrm{rot}}\vec{B}$$

$$\vec{\mathrm{grad}}(\vec{A}\cdot\vec{B}) = (\vec{A}\cdot\vec{\mathrm{grad}})\vec{B} + \vec{A}\wedge\vec{\mathrm{rot}}\vec{B} + (\vec{B}\cdot\vec{\mathrm{grad}})\vec{A} + \vec{B}\wedge\vec{\mathrm{rot}}\vec{A}$$

$$\nabla(\vec{F}\cdot\vec{F}) = 2(\vec{F}\cdot\nabla)\vec{F} + 2\vec{F}\wedge(\vec{\mathrm{rot}}\vec{F})$$

$$\mathrm{div}(\vec{X_0}\wedge\vec{B}) = -\vec{X_0}\cdot\vec{\mathrm{rot}}\vec{B}$$

$$\mathrm{div}(\vec{A}\wedge\vec{B}) = -\vec{A}\cdot\vec{\mathrm{rot}}\vec{B} + \vec{B}\cdot\vec{\mathrm{rot}}\vec{A}$$

$$\vec{\mathrm{rot}}(\vec{X_0}\wedge\vec{B}) = \vec{X_0}\cdot\mathrm{div}\vec{B} - (\vec{X_0}\cdot\vec{\mathrm{grad}})\vec{B}$$

$$\vec{\mathrm{rot}}(\vec{A}\wedge\vec{B}) = \vec{A}\cdot\mathrm{div}\vec{B} - (\vec{A}\cdot\vec{\mathrm{grad}})\vec{B} - \vec{B}\cdot\mathrm{div}\vec{A} + (\vec{B}\cdot\vec{\mathrm{grad}})\vec{A}$$

$$\vec{\mathrm{grad}}(fg) = f\cdot\vec{\mathrm{grad}}(g) + g\cdot\vec{\mathrm{grad}}(f)$$

$$\mathrm{div}(\rho \cdot \vec{V}) = \rho \cdot \mathrm{div}\vec{V} + \vec{\mathrm{grad}}(\rho) \cdot \vec{V}$$

$$\vec{\mathrm{rot}}(\rho \cdot \vec{V}) = \rho \cdot \vec{\mathrm{rot}}\vec{V} + \vec{\mathrm{grad}}(\rho) \wedge \vec{V}$$

$$\Delta(f \cdot g) = f \cdot \Delta g + 2\vec{\mathrm{grad}}(f) \cdot \vec{\mathrm{grad}}(g) + g \cdot \Delta f$$

$$\mathrm{div}(f \cdot \vec{\mathrm{grad}}(g) - g \cdot \vec{\mathrm{grad}}(f)) = f\Delta g - g\Delta f$$